

Thermodynamics of general scalar-tensor theory with non-minimally derivative coupling

Yumei Huang^{1,2,*} and Yungui Gong^{3,†}

¹*Institute of Physics and Electric Engineering,*

Mianyang Normal University, Mianyang 610021, China

²*Department of Astronomy, Beijing Normal University, Beijing 100875, China*

³*School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China*

Abstract

With the usual definitions for the entropy and the temperature associated with the apparent horizon, we discuss the first law of the thermodynamics on the apparent in the general scalar-tensor theory of gravity with the kinetic term of the scalar field non-minimally coupling to Einstein tensor. We show the equivalence between the first law of thermodynamics on the apparent horizon and Friedmann equation for the general models, by using a mass-like function which is equal to the Misner-Sharp mass on the apparent horizon. The results further support the universal relationship between the first law of thermodynamics and Friedmann equation.

PACS numbers: 04.50.Kd, 04.70.Dy, 98.80.-k

* huangyme@gmail.com

† yggong@mail.hust.edu.cn

I. INTRODUCTION

The area law of the entropy [1] and the Hawking radiation [2] of black holes may be clues to quantum gravity [3]. The Bekenstein-Hawking entropy of black holes is equal to one quarter of the area of the event horizon measured in Planck units [1, 2]. In 1981, Bekenstein extended the area law of the entropy of black holes to a weakly self-gravitating physical system in an asymptotically flat space-time, and he proposed the existence of a universal entropy bound [4]. These ideas were further generalized to the proposal of the holographic principle [5–7]. The holographic principle was later realized by the AdS/CFT correspondence which relates a gravitational theory in d -dimensional anti-de Sitter space with a conformal field theory living in a $(d - 1)$ -dimensional boundary space [8]. The AdS/CFT was widely applied to the study of holographic superconductors [9].

Black holes are the classical solutions of Einstein equation, so the four laws of black holes [10] may show a deep connection between gravitation and thermodynamics. For more general space-time, one may ask whether the connection still exists. It was shown that we can derive Einstein equation from the first law of thermodynamics by assuming the area law of the entropy for all local acceleration horizons [11]. In particular, for cosmological solutions with Friedmann-Robertson-Walker (FRW) metric, we also expect the equivalence between the first law of thermodynamics and Friedmann equations, and the relationship was indeed derived on the apparent horizon [12–16]. For more general theories of gravity, the universal relationship between thermodynamics and gravitation was discussed extensively [11–31]. For Brans-Dicke theory [32] and $f(R)$ gravity, a mass-like function was introduced to keep the equilibrium first law of thermodynamics on the apparent horizon [16].

Brans-Dicke theory is the simplest generalization of Einstein's general relativity. The most general scalar-tensor theory of gravity with equation of motion which contains no more than second time derivatives in four dimensional space-time is Horndeski theory [33]. The Lagrangian of Horndeski theory is

$$L_H = L_2 + L_3 + L_4 + L_5, \quad (1)$$

where $L_2 = K(\phi, X)$, $L_3 = -G_3(\phi, X)\Box\phi$,

$$L_4 = G_4(\phi, X)R + G_{4,X} [(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)],$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)], \quad (2)$$

$X = -\nabla_\mu\phi\nabla^\mu\phi/2$, $\Box\phi = \nabla_\alpha\nabla^\alpha\phi$, the functions K , G_3 , G_4 and G_5 are arbitrary functions of ϕ and X , and $G_{4,X}(\phi, X) = dG_4(\phi, X)/dX$. The general non-minimal coupling $F(\phi)R$ is included in L_4 if we choose $G_4(\phi, X) = F(\phi)$. The non-minimally derivative coupling with the kinetic term coupling to Einstein tensor is included in L_5 if we choose $G_5(\phi, X) = \phi$ [34, 35]. The scalar-tensor theory with the non-minimally derivative coupling $\omega^2 G^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ was discussed by lots of researchers recently [36–82].

In this paper, we discuss the thermodynamics of the general scalar-tensor theory of gravity with non-minimally derivative coupling. The paper is organized as follows. In sect. II, we review the scalar-tensor theory with non-minimally derivative coupling and discuss the relation between the first law of thermodynamics on the apparent horizon and Friedmann equation, and conclusions are drawn in Sect. III.

II. THE FIRST LAW OF THERMODYNAMICS

In this paper, we consider the general scalar-tensor theory of gravity with non-minimally derivative coupling

$$S = \int d^4x \sqrt{-g} \left[\frac{F(\phi)}{16\pi G} R - \frac{1}{2}(g^{\mu\nu} - \omega^2 G^{\mu\nu})\partial_\mu\phi\partial_\nu\phi - V(\phi) \right] + S_b, \quad (3)$$

where $V(\phi)$ is the potential of the scalar field, the derivative coupling constant ω has the dimension of inverse mass, S_b is the action for the matter, and the general non-minimal coupling $F(\phi)$ is an arbitrary function. For Brans-Dicke theory, $F(\phi) = \phi$ [32]. For more general non-minimal coupling, we usually choose $F(\phi) = 1 + \xi\phi^2$, and the special coupling $\xi = -1/6$ corresponds to the conformal coupling [83].

Taking variations of the action (3) with respect to the metric $g_{\mu\nu}$ leads to the field equation,

$$F(\phi)G_{\mu\nu} = 8\pi G(T_{\mu\nu}^b + T_{\mu\nu}^\phi) + \nabla_\mu\partial_\nu F(\phi) - g_{\mu\nu}\Box F(\phi), \quad (4)$$

where $\square = g^{\mu\nu}\nabla_\mu\nabla_\nu$, the energy-momentum tensor of the scalar field

$$\begin{aligned}
T_{\mu\nu}^\phi = & \left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}(\phi_{,\alpha})^2 - g_{\mu\nu}V(\phi) \right] \\
& - \omega^2 \left\{ -\frac{1}{2}\phi_{,\mu}\phi_{,\nu}R + 2\phi_{,\alpha}\nabla_{(\mu}\phi R_{\nu)}^\alpha + \phi^{,\alpha}\phi^{,\beta}R_{\mu\alpha\nu\beta} \right. \\
& + \nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla_\alpha\phi - \nabla_\mu\nabla_\nu\phi\square\phi - \frac{1}{2}(\phi_{,\alpha})^2G_{\mu\nu} \\
& \left. + g_{\mu\nu} \left[-\frac{1}{2}\nabla^\alpha\nabla^\beta\phi\nabla_\alpha\nabla_\beta\phi + \frac{1}{2}(\square\phi)^2 - \phi_{,\alpha}\phi_{,\beta}R^{\alpha\beta} \right] \right\}, \tag{5}
\end{aligned}$$

$T_{\mu\nu}^b$ is the energy-momentum tensor of the matter field, and the total energy-momentum tensor is

$$T_{\mu\nu} = T_{\mu\nu}^b + T_{\mu\nu}^\phi. \tag{6}$$

To discuss the cosmological evolution, we take the homogeneous and isotropic FRW metric

$$ds^2 = -dt^2 + \frac{a(t)^2}{1 - kr^2}dr^2 + a(t)^2r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{7}$$

where $k = 0, -1, +1$ represents a flat, open, and closed universe respectively. For convenience, we write the metric as the general form

$$ds^2 = g_{ab}dx^a dx^b + \tilde{r}^2 d\Omega^2, \tag{8}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ and $\tilde{r} = a(t)r$. For the FRW metric, $g_{tt} = -1$ and $g_{rr} = a^2(t)/(1 - kr^2)$. With this general metric, the ab components of Einstein tensor becomes

$$G_{ab} = -[2\tilde{r}\tilde{r}_{;ab} + g_{ab}(1 - g^{cd}\tilde{r}_{,c}\tilde{r}_{,d} - 2\tilde{r}\square\tilde{r})]/\tilde{r}^2, \tag{9}$$

where the covariant derivative is with respect to the metric g_{ab} and $\square = g^{ab}\nabla_a\nabla_b$. Since

$$\square F = \square F + 2\tilde{r}^{-1}\tilde{r}^{,a}F_{,a}, \tag{10}$$

eq. (4) becomes

$$\begin{aligned}
F(\phi)G_{ab} = & -F(\phi)[2\tilde{r}\tilde{r}_{;ab} + g_{ab}(1 - g^{cd}\tilde{r}_{,c}\tilde{r}_{,d} - 2\tilde{r}\square\tilde{r})]/\tilde{r}^2 \\
= & 8\pi GT_{ab} + \nabla_a\partial_b F(\phi) - g_{ab}\square F(\phi) - 2g_{ab}\tilde{r}^{-1}\tilde{r}^{,c}F_{,c}, \tag{11}
\end{aligned}$$

so

$$\begin{aligned}
2\tilde{r}\tilde{r}_{;ab} + g_{ab}(1 - g^{ab}\tilde{r}_{,a}\tilde{r}_{,b} - 2\tilde{r}\square\tilde{r}) = & -\frac{\tilde{r}^2}{F} [8\pi GT_{ab} + \nabla_a\partial_b F(\phi) - g_{ab}\square F(\phi)] \\
& + 2g_{ab}\tilde{r}\tilde{r}^{,c}\frac{F_{,c}}{F}. \tag{12}
\end{aligned}$$

After contraction, we get

$$2\tilde{r}\square\tilde{r} = \frac{\tilde{r}^2}{F} \left[8\pi GT - \square F(\phi) - \frac{4}{\tilde{r}} \tilde{r}^{,c} F_{,c} \right] + 2(1 - g^{ab}\tilde{r}_{,a}\tilde{r}_{,b}). \quad (13)$$

Substituting eq. (13) into eq. (12), we get

$$2\tilde{r}\tilde{r}_{;ab} = 8\pi G \frac{\tilde{r}^2}{F} (g_{ab}T - T_{ab}) - 2g_{ab}\tilde{r}\tilde{r}^{,c} \frac{F_{,c}}{F} - \frac{\tilde{r}^2}{F} \nabla_a \partial_b F(\phi) + g_{ab}(1 - g^{cd}\tilde{r}_{,c}\tilde{r}_{,d}). \quad (14)$$

For the FRW metric (7) and a perfect fluid for the source $T_{\mu\nu}$, eq. (14) reduces to the cosmological equations,

$$3F \left(H^2 + \frac{k}{a^2} \right) = 8\pi G\rho - 3H\dot{F}, \quad (15)$$

$$2F \left(\dot{H} - \frac{k}{a^2} \right) = -8\pi G(\rho + p) + H\dot{F} - \ddot{F}, \quad (16)$$

where the energy density ρ and the pressure p are

$$\rho = \rho_b + \frac{\dot{\phi}^2}{2}(1 + 9\omega^2 H^2) + V(\phi), \quad (17)$$

$$p = p_b + \frac{\dot{\phi}^2}{2} \left[1 - \omega^2 \left(2\dot{H} + 3H^2 + \frac{4H\ddot{\phi}}{\dot{\phi}} \right) \right] - V(\phi). \quad (18)$$

Comparing eqs. (17) and (18) with the standard Friedmann equations, we can think the extra terms as the energy density and pressure due to the non-minimal coupling $F(\phi)$,

$$\rho_F = -\frac{3H\dot{F}}{8\pi G}, \quad p_F = \frac{\ddot{F}}{8\pi G} + \frac{H\dot{F}}{4\pi G}. \quad (19)$$

With the FRW metric, the energy conservation equation becomes

$$8\pi G [\dot{\rho} + 3H(\rho + p)] = 3\dot{F} \left(2H^2 + \dot{H} + \frac{k}{a^2} \right). \quad (20)$$

In terms of the total energy $\rho_t = \rho + \rho_F$ and pressure $p_t = p + p_F$, we get

$$8\pi G [\dot{\rho}_t + 3H(\rho_t + p_t)] = 3\dot{F} \left(H^2 + \frac{k}{a^2} \right). \quad (21)$$

Now let us discuss the first law of thermodynamics on the apparent horizon. The apparent horizon is defined as $h = g^{ab}\tilde{r}_{,a}\tilde{r}_{,b} = 0$, so

$$\tilde{r}_A = ar_A = (H^2 + k/a^2)^{-1/2}. \quad (22)$$

Take the time derivative of the apparent horizon \tilde{r}_A , we get

$$\dot{\tilde{r}}_A = -\tilde{r}_A^3 H \left(\dot{H} - \frac{k}{a^2} \right). \quad (23)$$

Take the future directed ingoing null vector field $k^a = (1, -Hr)$ which is also the (approximate) generator of the horizon, and the Misner-Sharp mass

$$\mathcal{M} = \frac{F(\phi)}{2G} \tilde{r} (1 - g^{ab} \tilde{r}_{,a} \tilde{r}_{,b}) = \frac{F(\phi)}{2G} \frac{\tilde{r}^3}{\tilde{r}_A^2}, \quad (24)$$

since $k^a \tilde{r}_{,a} = 0$, we get

$$k^a \mathcal{M}_{,a} = \mathcal{M}_{,t} - Hr \mathcal{M}_{,r} = \frac{F_{,t}}{2G} \frac{\tilde{r}^3}{\tilde{r}_A^2} + \frac{F}{G} \tilde{r}^3 \left(-\frac{\dot{\tilde{r}}_A}{\tilde{r}_A^3} \right). \quad (25)$$

Therefore, on the apparent horizon, we find

$$-d\mathcal{E} = -k^c \mathcal{M}_{,c} dt = -\frac{dF}{2G} \tilde{r}_A + \frac{F(\phi)}{G} d\tilde{r}_A \quad (26)$$

On the other hand, if we choose the horizon temperature $T_A = 1/(2\pi\tilde{r}_A)$ and the horizon entropy $S_A = \pi\tilde{r}_A^2 F(\phi)/G$, then we get

$$\begin{aligned} T_A dS_A &= \frac{1}{2\pi G \tilde{r}_A} [2\pi F(\phi) \tilde{r}_A d\tilde{r}_A + \pi \tilde{r}_A^2 dF] \\ &= \frac{F(\phi)}{G} d\tilde{r}_A + \frac{dF}{2G} \tilde{r}_A \neq -d\mathcal{E} \end{aligned} \quad (27)$$

As expected, with the usual definitions of the temperature $T_A = 1/(2\pi\tilde{r}_A)$, the entropy $S_A = \pi\tilde{r}_A^2 F(\phi)/G$ associated with the apparent horizon, and the Misner-Sharp mass, the first law of thermodynamics on the apparent horizon does not hold for general scalar-tensor theories of gravity [14]. Without the non-minimal coupling $F(\phi)$, the first law of thermodynamics on the apparent horizon holds for the non-minimally derivative coupling [31]. To overcome the problem arising from the general non-minimal coupling $F(\phi)R$, a mass-like function which is equal to the Misner-Sharp mass on the apparent horizon was introduced [16]. Therefore, following ref. [16], we use the mass-like function instead. The mass-like function M is defined as

$$M = \frac{F(\phi)}{2G} \tilde{r} (1 + g^{ab} \tilde{r}_{,a} \tilde{r}_{,b}) = \frac{F(\phi)}{G} \tilde{r} - \mathcal{M}. \quad (28)$$

On the apparent horizon, since $h = g^{ab} \tilde{r}_{,a} \tilde{r}_{,b} = 0$ and $k^a h_{,a} = 2\dot{\tilde{r}}_A/\tilde{r}_A^{-1}$, so $M = \mathcal{M} = F(\phi)\tilde{r}_A/(2G) = 4\pi\tilde{r}_A^3\rho/3$. By using the result (26), on the apparent horizon, we get

$$\begin{aligned} dE &= k^c M_{,c} dt = \frac{1}{G} k^c F_{,c} \tilde{r}_A dt - \frac{dF}{2G} \tilde{r}_A + \frac{F}{G} d\tilde{r}_A \\ &= \frac{dF}{2G} \tilde{r}_A + \frac{F}{G} d\tilde{r}_A = T_A dS_A \end{aligned} \quad (29)$$

With the help of the mass-like function M , we show that the first law of the thermodynamics on the apparent horizon holds for the general scalar-tensor theory of gravity with non-minimally derivative coupling. Note that all the variables in the above equations are geometric quantities, the above relation is just a geometric identity and it is always true in FRW metric. Next we need to show that the geometric quantities are related with physical variables and the identity is equivalent to the Friedmann equation. Due to the extra term in the right hand side of eq. (21), we expect the energy flow rate through the apparent horizon to be

$$\begin{aligned}\frac{dE}{dt} &= \frac{4\pi\tilde{r}_A^3}{3} \frac{3\dot{F}}{8\pi G\tilde{r}_A^2} + 4\pi\tilde{r}_A^3 H(\rho_t + p_t) \\ &= \frac{\tilde{r}_A}{2G} \dot{F} + \frac{\tilde{r}_A^3 H}{2G} \left[8\pi G(\rho + p) - H\dot{F} + \ddot{F} \right].\end{aligned}\quad (30)$$

For Einstein gravity with $F(\phi) = 1$, the above result recovers the standard relation $dE = 4\pi\tilde{r}_A^3 H(\rho + p)dt$.

From the definition of the mass-like function (28), we get

$$M_{,c} = \frac{(F\tilde{r})_{,c}}{G} - \frac{(F\tilde{r})_{,c}}{2G} (1 - g^{ab}\tilde{r}_{,a}\tilde{r}_{,b}) + \frac{F\tilde{r}}{G} (\tilde{r}^{,a}\tilde{r}_{;ac}). \quad (31)$$

Substituting the field equations (13) and (14) into eq. (31), we get

$$\begin{aligned}M_{,c} &= \frac{(F\tilde{r})_{,c}}{G} + 2\pi\tilde{r}^3 \frac{F_{,c}}{F} T - 4\pi\tilde{r}^2 (T_c^a - \delta_c^a T) \tilde{r}_{,a} - \frac{F_{,c}}{FG} \tilde{r}^2 \tilde{r}^{,a} F_{,a} - \frac{F_{,a}}{G} \tilde{r} \tilde{r}^{,a} \tilde{r}_{,c} \\ &\quad - \frac{F_{,c}}{2G} \tilde{r}^2 \square \tilde{r} - \frac{F_{,c}}{4FG} \tilde{r}^3 \square F - \frac{F_{;ac}}{2G} \tilde{r}^2 \tilde{r}^{,a}.\end{aligned}\quad (32)$$

By using the field equation (32), we get the energy flow through the apparent horizon

$$dE = k^c M_{,c} dt = \frac{dF}{2G} \tilde{r}_A + \frac{\tilde{r}_A^3 H}{2G} \left[8\pi G(\rho + p) - H\dot{F} + \ddot{F} \right] dt. \quad (33)$$

This is exactly what we speculate in eq. (30). Combining eqs. (16) and (23), we get

$$\dot{\tilde{r}}_A = \frac{\tilde{r}_A^3 H}{2F} \left[8\pi G(\rho + p) - H\dot{F} + \ddot{F} \right]. \quad (34)$$

Substituting eq. (34) into eq. (33), we obtain

$$dE = \frac{dF}{2G} \tilde{r}_A + \frac{F}{G} d\tilde{r}_A = T_A dS_A. \quad (35)$$

Therefore, the first law of thermodynamics on the apparent horizon is derived from Friedmann equation. To derive Friedmann equation from the first law of thermodynamics on the apparent horizon (35), we combine eqs. (30) and (35), then we get

$$F\dot{\tilde{r}}_A = \frac{\tilde{r}_A^3 H}{2} \left[8\pi G(\rho + p) - H\dot{F} + \ddot{F} \right]. \quad (36)$$

Substitute eq. (23) into the above equation, we obtain eq. (16) from the first law of thermodynamics on the apparent horizon. Combining the energy conservation equation (20) and eq. (16), after integration, we finally obtain the Friedmann equation (15). With help of the mass-like function (28) and the proposed energy flow through the apparent horizon (30), we show the equivalence between the first law of thermodynamics on the apparent horizon and Friedmann equation.

III. CONCLUSIONS

With the help of the mass-like function which is equal to the Misner-Sharp mass on the apparent horizon proposed in ref. [16], we show that the first law of thermodynamics on the apparent horizon is a geometric identity for the general scalar-tensor theory of gravity with non-minimally derivative coupling. To discuss the equivalence between the first law of thermodynamics on the apparent horizon and Friedmann equation, we need to connect the mass-like function with the physical energy flow through the apparent horizon. In standard cosmology, the energy is conserved, i.e., $\dot{\rho}_b + 3H(\rho_b + p_b) = 0$, and the energy flow through the apparent horizon is $4\pi\tilde{r}_A^3 H(\rho_b + p_b)$. In general scalar-tensor theories of gravity, we also expect that the energy flow through the apparent horizon has the term $4\pi\tilde{r}_A^3 H(\rho_t + p_t)$. However, in the general scalar-tensor theory of gravity with non-minimally derivative coupling, the total energy is not conserved, we have extra contribution coming from the non-minimal coupling $F(\phi)$, so we propose that the energy flow through the apparent horizon is given by eq. (30). By using Friedmann equations and the definition of the mass-like function, we show that eq. (30) gives the energy flow through the apparent horizon. Then, we show the equivalence between Friedmann equation and the first law of thermodynamics on the apparent horizon. Therefore, our results further support the universal relationship between the first law of thermodynamics and Friedmann equations.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 11175270 and 11475065) and the Program for New Century Excellent Talents in

- [1] Bekenstein J D. Black holes and entropy. *Phys Rev D*, 1973, 7: 2333–2346
- [2] Hawking S. Particle Creation by Black Holes. *Commun Math Phys*, 1975, 43: 199–220
- [3] Wald R M. The thermodynamics of black holes. *Living Rev Rel*, 2001, 4: 6
- [4] Bekenstein J D. A Universal Upper Bound on the Entropy to Energy Ratio for Bounded Systems. *Phys Rev D*, 1981, 23: 287
- [5] 't Hooft G. Dimensional reduction in quantum gravity. in *Salamfest, 1993*: 0284–296, arXiv: gr-qc/9310026
- [6] Susskind L. The World as a hologram. *J Math Phys*, 1995, 36: 6377–6396
- [7] Witten E. Anti-de Sitter space and holography. *Adv Theor Math Phys*, 1998, 2: 253–291
- [8] Maldacena J M. The Large N limit of superconformal field theories and supergravity. *Adv Theor Math Phys*, 1998, 2: 231
- [9] Cai R G, Li L, Li L F, et al. Introduction to Holographic Superconductor Models. *Sci China Phys Mech Astron*, 2015, 58(6): 060401
- [10] Bardeen J M, Carter B, Hawking S. The Four laws of black hole mechanics. *Commun Math Phys*, 1973, 31: 161–170
- [11] Jacobson T. Thermodynamics of space-time: The Einstein equation of state. *Phys Rev Lett*, 1995, 75: 1260–1263
- [12] Cai R G, Kim S P. First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe. *JHEP*, 2005, 0502: 050
- [13] Akbar M, Cai R G. Thermodynamic Behavior of Friedmann Equations at Apparent Horizon of FRW Universe. *Phys Rev D*, 2007, 75: 084003
- [14] Cai R G, Cao L M. Unified first law and thermodynamics of apparent horizon in FRW universe. *Phys Rev D*, 2007, 75: 064008
- [15] Gong Y, Wang B, Wang A. Thermodynamical properties of the Universe with dark energy. *JCAP*, 2007, 0701: 024
- [16] Gong Y, Wang A. The Friedmann equations and thermodynamics of apparent horizons. *Phys Rev Lett*, 2007, 99: 211301

- [17] Hayward S A. Unified first law of black hole dynamics and relativistic thermodynamics. *Class Quant Grav*, 1998, 15: 3147–3162
- [18] Padmanabhan T. Classical and quantum thermodynamics of horizons in spherically symmetric space-times. *Class Quant Grav*, 2002, 19: 5387–5408
- [19] Padmanabhan T. Gravity and the thermodynamics of horizons. *Phys Rept*, 2005, 406: 49–125
- [20] Wang B, Gong Y, Abdalla E. Thermodynamics of an accelerated expanding universe. *Phys Rev D*, 2006, 74: 083520
- [21] Eling C, Guedens R, Jacobson T. Non-equilibrium thermodynamics of spacetime. *Phys Rev Lett*, 2006, 96: 121301
- [22] Kothawala D, Sarkar S, Padmanabhan T. Einstein’s equations as a thermodynamic identity: The Cases of stationary axisymmetric horizons and evolving spherically symmetric horizons. *Phys Lett B*, 2007, 652: 338–342
- [23] Zhou J, Wang B, Gong Y, et al. The Second law of thermodynamics in the accelerating universe. *Phys Lett B*, 2007, 652: 86–91
- [24] Gong Y, Wang B, Wang A. On thermodynamical properties of dark energy. *Phys Rev D*, 2007, 75: 123516
- [25] Cai R G, Cao L M, Hu Y P. Hawking Radiation of Apparent Horizon in a FRW Universe. *Class Quant Grav*, 2009, 26: 155018
- [26] Hayward S, Di Criscienzo R, Vanzo L, et al. Local Hawking temperature for dynamical black holes. *Class Quant Grav*, 2009, 26: 062001
- [27] Padmanabhan T. Thermodynamical Aspects of Gravity: New insights. *Rept Prog Phys*, 2010, 73: 046901
- [28] Sharif M, Zubair M. Thermodynamics in $f(R,T)$ Theory of Gravity. *JCAP*, 2012, 1203: 028
- [29] Mitra S, Saha S, Chakraborty S. A Study of Universal Thermodynamics in Brane World Scenario. *Adv High Energy Phys*, 2015, 2015: 430764
- [30] Helou A. Dynamics of the Cosmological Apparent Horizon: Surface Gravity & Temperature. *arXiv*: 1502.04235
- [31] Huang Y, Gong Y, Liang D, et al. Thermodynamics of scalar-tensor theory with non-minimally derivative coupling. *Eur Phys J C*, 2015, 75(7): 351
- [32] Brans C, Dicke R. Mach’s principle and a relativistic theory of gravitation. *Phys Rev*, 1961, 124: 925–935

- [33] Horndeski G W. Second-order scalar-tensor field equations in a four-dimensional space. *Int J Theor Phys*, 1974, 10: 363–384
- [34] Sushkov S V. Exact cosmological solutions with nonminimal derivative coupling. *Phys Rev D*, 2009, 80: 103505
- [35] Germani C, Kehagias A. New Model of Inflation with Non-minimal Derivative Coupling of Standard Model Higgs Boson to Gravity. *Phys Rev Lett*, 2010, 105: 011302
- [36] Amendola L. Cosmology with nonminimal derivative couplings. *Phys Lett B*, 1993, 301: 175–182
- [37] Capozziello S, Lambiase G. Nonminimal derivative coupling and the recovering of cosmological constant. *Gen Rel Grav*, 1999, 31: 1005–1014
- [38] Capozziello S, Lambiase G, Schmidt H. Nonminimal derivative couplings and inflation in generalized theories of gravity. *Annalen Phys*, 2000, 9: 39–48
- [39] Daniel S F, Caldwell R R. Consequences of a cosmic scalar with kinetic coupling to curvature. *Class Quant Grav*, 2007, 24: 5573–5580
- [40] Germani C, Martucci L, Moyassari P. Introducing the Slotheon: a slow Galileon scalar field in curved space-time. *Phys Rev D*, 2012, 85: 103501
- [41] Germani C, Kehagias A. Cosmological Perturbations in the New Higgs Inflation. *JCAP*, 2010, 1005: 019
- [42] Germani C, Watanabe Y. UV-protected (Natural) Inflation: Primordial Fluctuations and non-Gaussian Features. *JCAP*, 2011, 1107: 031
- [43] Germani C, Watanabe Y, Wintergerst N. Self-unitarization of New Higgs Inflation and compatibility with Planck and BICEP2 data. *JCAP*, 2014, 1412(12): 009
- [44] Tsujikawa S. Observational tests of inflation with a field derivative coupling to gravity. *Phys Rev D*, 2012, 85: 083518
- [45] Sadjadi H M, Goodarzi P. Oscillatory inflation in non-minimal derivative coupling model. *Phys Lett B*, 2014, 732: 278–284
- [46] Saridakis E N, Sushkov S V. Quintessence and phantom cosmology with non-minimal derivative coupling. *Phys Rev D*, 2010, 81: 083510
- [47] Sushkov S. Realistic cosmological scenario with non-minimal kinetic coupling. *Phys Rev D*, 2012, 85: 123520

- [48] Skugoreva M A, Sushkov S V, Toporensky A V. Cosmology with nonminimal kinetic coupling and a power-law potential. *Phys Rev D*, 2013, 88(10): 083539
- [49] De Felice A, Tsujikawa S. Inflationary non-Gaussianities in the most general second-order scalar-tensor theories. *Phys Rev D*, 2011, 84: 083504
- [50] Sadjadi H M. Super-acceleration in non-minimal derivative coupling model. *Phys Rev D*, 2011, 83: 107301
- [51] Sadjadi H M. Rapid Oscillatory quintessence with variable equation of state parameter in non minimal derivative coupling model. *Gen Rel Grav*, 2014, 46(11): 1817
- [52] Minamitsuji M. Solutions in the scalar-tensor theory with nonminimal derivative coupling. *Phys Rev D*, 2014, 89(6): 064017
- [53] Granda L. Non-minimal Kinetic coupling to gravity and accelerated expansion. *JCAP*, 2010, 1007: 006
- [54] Granda L, Cardona W. General Non-minimal Kinetic coupling to gravity. *JCAP*, 2010, 1007: 021
- [55] Granda L. Dark energy from scalar field with Gauss Bonnet and non-minimal kinetic coupling. *Mod Phys Lett A*, 2012, 27: 1250018
- [56] de Rham C, Heisenberg L. Cosmology of the Galileon from Massive Gravity. *Phys Rev D*, 2011, 84: 043503
- [57] Jinno R, Mukaida K, Nakayama K. The universe dominated by oscillating scalar with non-minimal derivative coupling to gravity. *JCAP*, 2014, 1401: 031
- [58] Sami M, Shahalam M, Skugoreva M, et al. Cosmological dynamics of non-minimally coupled scalar field system and its late time cosmic relevance. *Phys Rev D*, 2012, 86: 103532
- [59] Anabalón A, Cisterna A, Oliva J. Asymptotically locally AdS and flat black holes in Horndeski theory. *Phys Rev D*, 2014, 89(8): 084050
- [60] Rinaldi M. Black holes with non-minimal derivative coupling. *Phys Rev D*, 2012, 86: 084048
- [61] Koutsoumbas G, Ntrekis K, Papantonopoulos E. Gravitational Particle Production in Gravity Theories with Non-minimal Derivative Couplings. *JCAP*, 2013, 1308: 027
- [62] Cisterna A, Erices C. Asymptotically locally AdS and flat black holes in the presence of an electric field in the Horndeski scenario. *Phys Rev D*, 2014, 89(8): 084038
- [63] Huang Y, Gao Q, Gong Y. The Phase-space analysis of scalar fields with non-minimally derivative coupling. *Eur J Phys C*, 2015, 75: 143

- [64] Bravo-Gaete M, Hassaine M. Lifshitz black holes with a time-dependent scalar field in a Horndeski theory. *Phys Rev D*, 2014, 89(10): 104028
- [65] Bravo-Gaete M, Hassaine M. Thermodynamics of a BTZ black hole solution with an Horndeski source. *Phys Rev D*, 2014, 90(2): 024008
- [66] Bruneton J P, Rinaldi M, Kanfon A, et al. Fab Four: When John and George play gravitation and cosmology. *Adv Astron*, 2012, 2012: 430694
- [67] Feng K, Qiu T, Piao Y S. Curvaton with nonminimal derivative coupling to gravity. *Phys Lett B*, 2014, 729: 99–107
- [68] Feng K, Qiu T. Curvaton with nonminimal derivative coupling to gravity: Full perturbation analysis. *Phys Rev D*, 2014, 90(12): 123508
- [69] Heisenberg L, Kimura R, Yamamoto K. Cosmology of the proxy theory to massive gravity. *Phys Rev D*, 2014, 89(10): 103008
- [70] Sadjadi H M, Goodarzi P. Temperature in warm inflation in non minimal kinetic coupling model. *Eur Phys J C*, 2015, 75(10): 513
- [71] Cisterna A, Delsate T, Rinaldi M. Neutron stars in general second order scalar-tensor theory: The case of nonminimal derivative coupling. *Phys Rev D*, 2015, 92(4): 044050
- [72] Dalianis I, Farakos F. Higher Derivative D-term Inflation in New-minimal Supergravity. *Phys Lett B*, 2014, 736: 299–304
- [73] Dalianis I, Farakos F. Exponential potential for an inflaton with nonminimal kinetic coupling and its supergravity embedding. *Phys Rev D*, 2014, 90(8): 083512
- [74] Dalianis I, Farakos F. Non-minimal derivative couplings and inflation in supergravity. *PoS*, 2015, CORFU2014: 098
- [75] Ema Y, Jinno R, Mukaida K, et al. Particle Production after Inflation with Non-minimal Derivative Coupling to Gravity. *JCAP*, 2015, 1510(10): 020
- [76] Aoki S, Yamada Y. Impacts of supersymmetric higher derivative terms on inflation models in supergravity. *JCAP*, 2015, 1507(07): 020
- [77] Yang N, Gao Q, Gong Y. Inflationary models with non-minimally derivative coupling. *arXiv*: 1504.05839
- [78] Yang N, Gao Q, Gong Y. Inflation with non-minimally derivative coupling. *Int J Mod Phys A*, 2015, 30(28 & 29): 1545004

- [79] Harko T, Lobo F S N, Mimoso J P, et al. Gravitational induced particle production through a nonminimal curvaturematter coupling. *Eur Phys J C*, 2015, 75: 386
- [80] Matsumoto J, Sushkov S V. Cosmology with nonminimal kinetic coupling and a Higgs-like potential. *JCAP*, 2015, 1511(11): 047
- [81] Zhu Y, Gong Y. PPN parameters in gravitational theory with non-minimally derivative coupling. *arXiv*: 1512.05555
- [82] Koutsoumbas G, Ntsekis K, Papantonopoulos E, et al. Gravitational Collapse in Horndeski Theory. *arXiv*: 1512.05934
- [83] Callan Jr Curtis G, Coleman S R, Jackiw R. A New improved energy - momentum tensor. *Annals Phys*, 1970, 59: 42–73